Note on the Development of Probabilities for Life Insurance Contracts

Once a life insurer decides on the mortality table that will be used to determine premiums for life insurance benefits, the insurer can set about developing calculations based on a set of probabilities. There are four basic probabilities that relate to these calculations.

The first has already been presented:

1qx = the probability a person age x dies before reaching age x+1

= l x – l x+1

l x

In general, we can also define:

tqx = the probability a person age x dies before reaching age x+t. as

= l x – l x+t

l x

and, since a person age x will either live or die between age x and x+t, it is possible to define

tpx = the probability a person age x lives to age x+t, so that

tqx + tpx = 1 which means that

tpx = 1 - tqx = 1 - [ ( l x – l x+t  )/ l x )] = l x  - (l x – l x+t )

l x l x

so, tpx  = l x+t  [the number of individuals who make it to age x+t]

l x  [the number of individuals who start out at age x]

The probability that a person age x lives to a particular age x+t [tpx ] can be combined with the present value interest factors of v, where v = 1/(1+i) to determine the net single premium on any number of life annuities. For example, if you were to consider finding the cost of a life annuity immediate to a person age x where the benefit amount was a $1 each year, the time line would be:

$1 $1 $1 . . .

| | | |

X X+1 X+2 X+3 . . . .

a x  = $1 v 1px + $1 v 2  2px + $1 v 3 3px + . . .

where this progression continues out to the end of the mortality table. The steps in computing this present value of the cost of a life annuity issued to a person age x with payments made at the end of each year based on survivorship are as follows: (1) specify an interest rate and compute the value of v (2) set of a column of ages beginning with age x, and a corresponding column of durations from 0 to the last payment date (3) set up another column of present value discount factors for each row with v being raised to the power found in the duration column for each payment date (4) load in another column by copying and pasting from a suitable mortality table the l x ‘s giving the number people living at each age x on the spreadsheet (5) develop another column of probabilities for tpx using the l x ‘s (5) create another column of discounted expected benefits found by multiplying the present value discount factor in each row times the probability that the person lives to age x+ t.

Examples of how to determine the cost of life annuities can be accessed through the lecture notes page of the insurance operations website under the title heading: Spreadsheet Examples of Annuity Calculations Involving Interest and Mortality Information

**Extra Credit Assignment Due: November 15, 2011 worth 15 points**

As an extra credit assignment, determine (1) the cost of a 15 year life annuity immediate issued to a person age 45 where the annual benefit amount is $10,000 and the interest rate is 4.5% (2) the cost of a 15 year life annuity due issued to a person age 45 where the annual benefit amount is $10,000 (3) which annuity is more expensive and why?